A Case Study on Asset Allocation in a Markowitz World

With extremely low interest rates, the task of efficient asset allocation is a challenging one. Michael Aichinger, Andreas Binder and Sophia Simmill move within the Markowitz world focusing on the multiperiod case.

With a global wealth of 256 trillion USD in 2016 (source: [3]) and with an ongoing regime of extremely low interest rates, the task of efficient asset allocation is a challenging one. In this article we move within the Markowitz world [4], focusing on the multiperiod case. While it is common sense (although sometimes falsified) that diversification should reduce risk with the desired return remaining unchanged, in our examples we wanted to quantify the volume of proposed reallocation under various trading strategies in the multiperiod case.

Assumptions on the tradable assets and on the trading restrictions

We assume that all tradable assets are equities, and that their joint movement is a multidimensional geometric Brownian motion, leading to a multidimensional log-normal distribution. We assume that no dividends are paid (or, alternatively, dividends are immediately reinvested in the same company).

At time t, let the investor possess a certain capital K_t that should be allocated to a pool of N assets in an efficient way under the following restrictions:

- All assets are equity shares. There is no risk-free investment available (at least not within the capital K_t).
- The entirety of the capital has to be allocated. This means that the weights of the different assets within the portfolio have to sum up to 1. In one of the example cases, short selling will be allowed. In such a case, if the investor short sells equities for 80 percent of the capital, she has to own (long) assets for 180 percent of the capital.
- Linear inequality constraints are allowed. Thus, the investor could restrict herself so that the weight of every asset should lie between, say, 0 and 10 percent, or the amount of Swiss equity should be below 20 percent, or the finance sector should have at least 15 percent.

Data used and benchmark

We used the time series of the $N = 50$ companies with the largest market capitalization on March 31, 2015, as reported by the Financial Times Global 500.[1] For these shares, we used the time series of closing prices for the three years between 2013 and 2015 (the 2013 data were used for estimation only). All stock quotes (different currencies) were converted to EUR. Assume that on January 1, 2014, the investor starts with 1 million EUR and allocates 2 percent of the then available capital to each of the 50 stocks at every trading day. Without transaction costs, the portfolio value would develop as shown in Figure 1.

In this benchmark example, the final portfolio value was 1.52 million EUR with a realized annualized volatility of 15.2 percent. To achieve the reallocation on each of the 504 trading days, the buying and selling volume (double counting the trading, leading to equal proportions again) was 7.289 million EUR. No transaction costs were taken into account.

In August 2015, the China shock[2] led to a massive rumble in the financial markets. This can be clearly observed in the drop after day 400.

Variance, covariance, and return estimates

To apply a Markowitz framework, we need estimates for the returns and for the variance–covariance matrix. In this article, at every business day, we use a moving window of 252 business days, without fading memory, to obtain a very simple estimate. Although the variance–covariance matrix obtained by this procedure is quite robust in the course of time (there is only one day dropping out and one new one coming in), this is not necessarily the case for the portfolio weights, as we will see in the sequel.

Figure 1: Benchmark portfolio (2 percent of capital in each asset): x-axis, number of trading day; y-axis, portfolio value

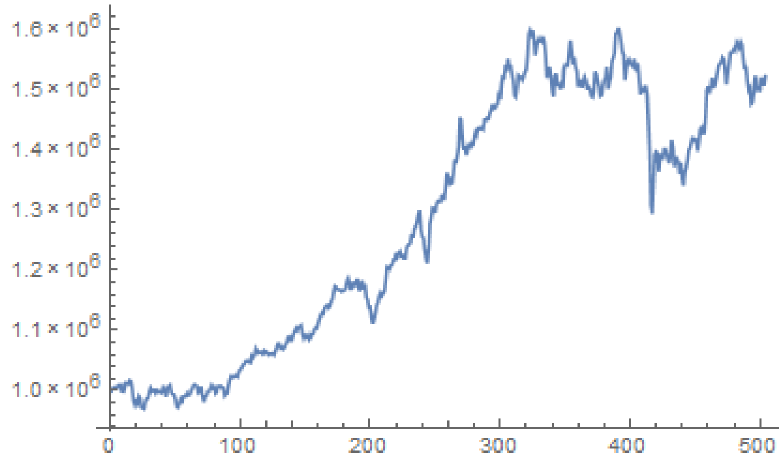
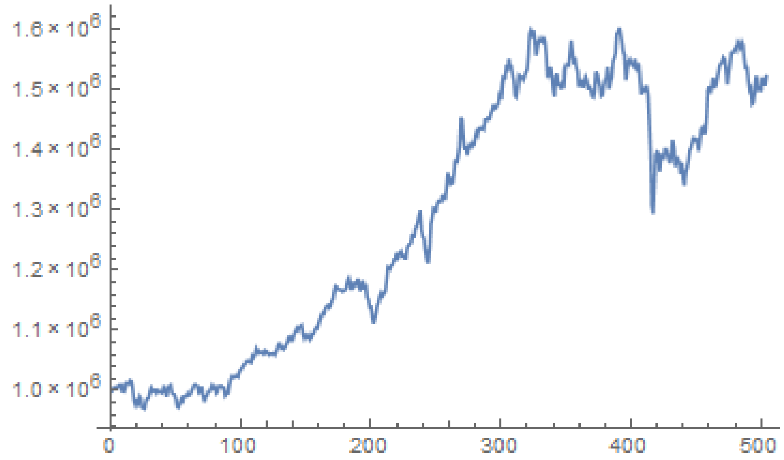


Figure 2: Long-only portfolio, minimal risk



A long-only portfolio with minimal Markowitz risk

The first trading strategy we take into account is the following one: on every business day, the investor reallocates her portfolio in such a way that the estimated variance of the portfolio (based on the variance–covariance as above) is minimized. There is no short selling allowed, and all capital must be spent in equity.

This is a quadratic minimization problem with linear inequality constrains [1, 2], which is solved by a proprietary modification of the interior point algorithm [5]; see also [6, 7].

The final capital in the portfolio of Figure 2 was 1.41 million EUR, with an annualized realized volatility of 11.85 percent. To achieve this, a volume of 30.34 million EUR (double counting) had to be traded.

Can short selling reduce risk?

Can this portfolio volatility be decreased further by allowing short selling (and still have full allocation)? As we have a quadratic objective function with positive eigenvalues of the Hessian, the portfolio weights (either positive or negative) cannot grow beyond all limits. Therefore, the only constraint that has to be satisfied is that the "sum of weights equals one," and there are no inequality constraints.

Indeed, it works out (see Figure 3). The final value of 1.4 million EUR is obtained with an annualized realized volatility of 9.97 percent. However, the trading volume was 122.93 million EUR.

High-risk regimes

What happens when we are ready to take higher risk? Let us consider two cases:

(a) Put all eggs into one basket only (i.e., invest the total of capital in the equity from which you expect – based on historical estimates – the highest return). See Figure 4.

(b) Similar to (a), but with five baskets: allocate 20 percent of capital to the five candidates that are expected to perform best ("per-

Figure 3: Short selling allowed, full allocation, minimized risk

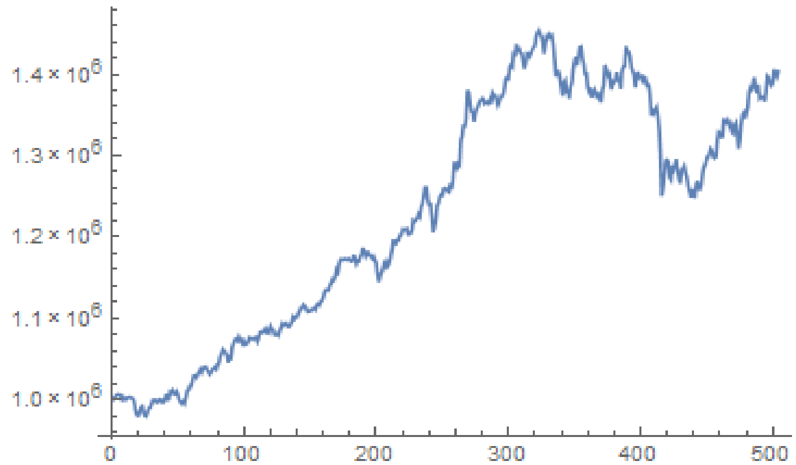


Figure 4: Invest in the best-performing equity

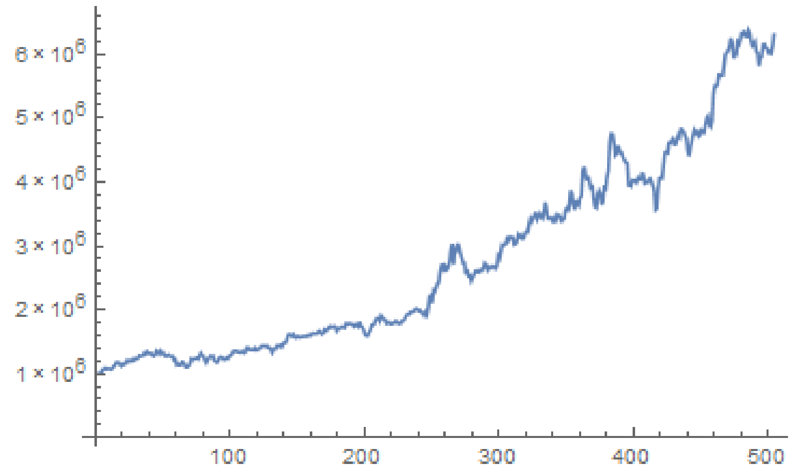


Figure 5: Invest in the five best-performing equities

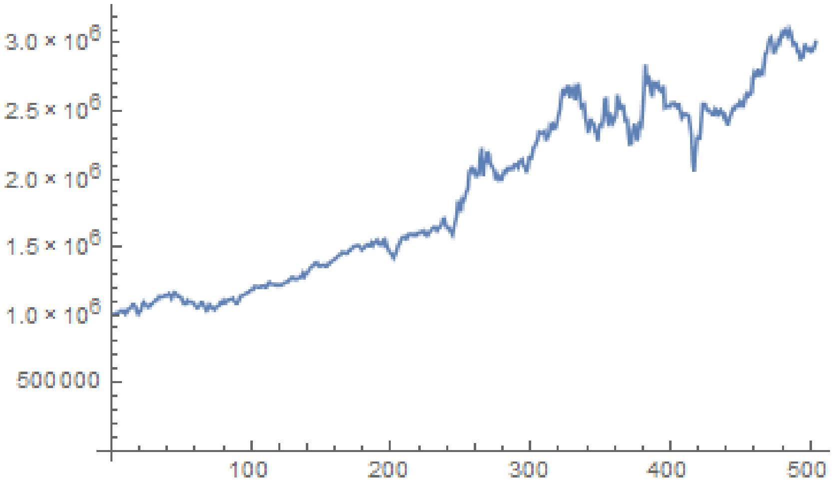
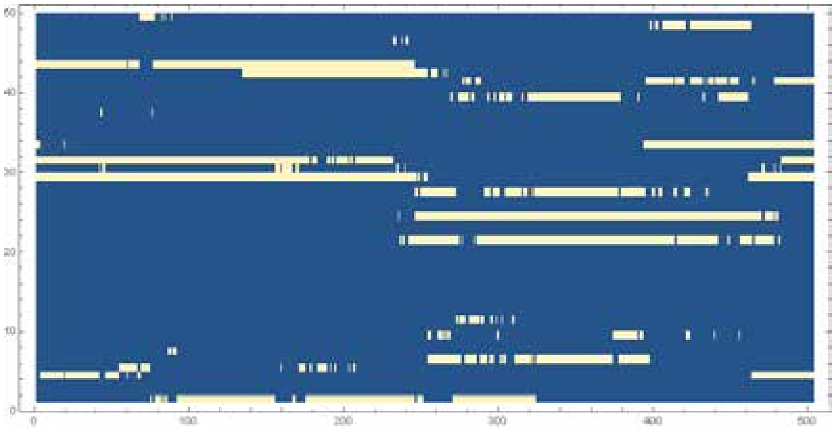


Figure 6: Asset allocation in the course of time; five stocks picked



form” again in the “high-return” sense). See Figure 5.

In practice, these strategies were implemented in a slightly different way, yielding the same outcome: if we take the portfolio with the highest expected return among all portfolios with equity weights between 0 and 1, then it is easy to prove that a single asset allocation (the one with the highest estimated return) is the solution. Similarly, when restricting to weights between 0 and 20 percent, the highest expected return is a five-equity strategy.

These strategies would have led to higher returns, to higher volatilities, and (by chance, not necessarily) to a higher trading volume. In Figure 4: final value = 6.31 million EUR, realized volatility = 39.99%, trading volume = 235.56 million EUR. In Figure 5: final value = 3.01 million EUR, realized volatility = 30.00%, trading volume = 129.27 million EUR.

Reducing the trading volume

It seems that the daily reallocation generates a lot of churning. Let us have a closer look at the asset allocation which led to the wealth development in Figure 5. The asset allocation is indicated in Figure 6.

It seems that the year 2014 (the first 250 days of Figure 6) was a good time for trend followers, with quite steady peak performers. However, in 2015 the pattern gets more rugged, and every time one stock is replaced by another, a trading volume of 40 percent is accounted for (20 percent sell, 20 percent buy). This might explain the high transaction volume.

Outlook

We have presented a case study of different asset-allocation schemes within a Markowitz world. Without taking into account transaction costs, the results are quite promising. In order to reduce the trading volume, stabilization procedures have to be applied. We can think of less frequent reallocation (weekly?), of regularizing the variance–covariance matrix (shifting the eigenvalues), or of procedures similar to those in [8] to obtain lower costs. This should be the topic of further investigations.

About the Authors

Michael Aichinger and Andreas Binder both have a strong background in quantitative modeling and risk management. They are the authors of the book *A Workout in Computational Finance*, published by Wiley. Sophia Simmill is a student of Economics and International Relations at the University of Aberdeen, UK, who did an internship with MathConsult in summer 2016.

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FOOTNOTES

1. These were: Apple, ExxonMobile, BerkshireHathaway, Google, Microsoft, Petrochina, WellsFargo, Johnson & Johnson, Industrial and Commercial Bank of China, Novartis, China Mobile, Walmart Stores, General Electric, Nestlé, Toyota, Roche, JPMorganChase, Procter & Gamble, Samsung Electronics, Pfizer, China Construction Bank, Verizon Communications, Chevron, Bank of China, AnheuserBuschInBev, RoyalDutchShell, Agricultural Bank of China, Oracle, Facebook, Walt Disney, Tencent, Coca-Cola, Amazon, AT&T, HSBC, Merck, Bank of America, IBM, ChinaLifeInsurance, Citigroup, Homedepot, Intel, Gileadsciences, Comcast, Pepsico, Ciscosystems, Sanofi, Visa, Volkswagen, and Bayer.
2. See, e.g., www.theguardian.com/business/2015/aug/12/china-yuan-slips-again-after-devaluation.

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